Seesaw Right Handed Neutrino as the Sterile Neutrino for LSND

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Abstract

We show that a double seesaw framework for neutrino masses with $\mu - \tau$ exchange symmetry can lead to one of the righthanded seesaw partners of the light neutrinos being massless. This can play the role of a light sterile neutrino, giving a $3 + 1$ model that explains the LSND results. We get a very economical scheme, which makes it possible to predict the full 4×4 neutrino mass matrix if CP is conserved. Once CP violation is included, effect of the LSND mass range sterile neutrino is to eliminate the lower bound on neutrinoless double beta decay rate which exists for the three neutrino case with inverted mass hierarchy. The same strategy can also be used to generate a natural $3 + 2$ model for LSND, which is also equally predictive for the CP conserving case in the limit of exact $\mu - \tau$ symmetry.

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I.   INTRODUCTION

Our knowledge of neutrinos has undergone a major revolution in the last seven years. There is now convincing evidence for nonzero neutrino masses and mixings. Most of the data seem to be consistent with a picture of three active neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ having mass and mixing among themselves. While two of the three mixings that characterize the three neutrino system i.e. θ_{12} corresponding to solar neutrino oscillation and θ_{23} corresponding to that for atmospheric neutrinos are fairly well determined, the third mixing angle θ_{13} has only an upper limit[1] and is the subject of many future experiments[2, 3].

In anticipation of the results from the planned experiments, there has been theoretical speculations about possible meaning of the existence of a small or "large" θ_{13}[4]. An interesting possibility which has been inspired by the experimental observation of near maximal atmospheric mixing angle (θ_{23}) and small upper limit on θ_{13} is that there may be an approximate $\mu - \tau$ interchange symmetry in the neutrino sector[5]. Even though large mass difference between the muon and the tau would appear to go against it, in supersymmetric theories where the muons and the neutrinos get mass from different Higgs fields, we may have a situation where an exact $\mu - \tau$ symmetry prevails at high scale but at lower scales it may only survive in the H_u sector but not in the H_d sector. Examples of this type exist in literature[6, 7]. The next generation of θ_{13} searches can throw light on this symmetry[8] making it an important idea to pursue. Approximate $\mu - \tau$ symmetry has also interesting implications for the origin of matter via leptogenesis[9]. In this paper, we discuss another application of the idea of $\mu - \tau$ symmetry for neutrinos.

While most of the present data have been discussed in the context of a three neutrino picture, the results of the LSND experiment[10], if confirmed by the Mini-BooNe[11] experiment currently in progress would require that there be one[12, 13] or two sterile neutrinos[14] that mix with the active ones. The sterile neutrinos must have a mass in the range of an eV, thereby posing a major challenge for theories since being standard model singlets the sterile neutrinos could in principle have a mass of order of the Planck scale. There exist a number of interesting proposals in literature to explain the lightness of the sterile neutrino[15] and in this paper we show that this can happen in the context of the conventional seesaw mechanism[16] which is anyway needed to understand the small masses of the known neutrinos.

Furthermore, with four neutrinos mixing with each other, it becomes difficult to make any predictions for the various mass matrix elements. Our model has the additional advantage that in the CP conserving case, the symmetries of the theory allow us to write down all the elements of the 4×4 mass matrix. For some examples where predictions for the four neutrino case have been attempted, see [17, 18].

The seesaw mechanism introduces three right handed neutrinos to the standard model. Since the right handed neutrinos are sterile with respect to known weak interactions, it has often been speculated as to whether one of them can somehow be ultralight and play the role of the sterile neutrino while at the same time maintaining the conventional seesaw mechanism for the active neutrinos. Clearly, the only way it can happen is if one can "turn off" the expected large mass of one of them without effect of the seesaw mechanism. The first example of a model where such a situation arises was proposed in Ref.[19] where the neutrino sector was assumed to obey an $(L_e - L_\mu - L_\tau) \otimes S_{2R}$ symmetry. In this case, one of the three right handed neutrinos becomes massless instead of being supermassive as in the usual seesaw case while the other two become superheavy. The latter become responsible for the smallness of the masses of two of the active neutrinos via seesaw. The third one does not need a superheavy seesaw partner since it also remain exactly massless as a consequence of the symmetry. The effect of the symmetry is therefore essentially to turn the 3×3 seesaw into a 2×2 one. Then turning on small deviations from this symmetry one can generate tiny mass for the ν_s as well as small mixings with ν_e and ν_μ. Using this idea, one could write models for both 3+1[19] as well as 2+2 [20] scenarios.

In this paper, we point out that in the context of a generalized seesaw model, a somewhat simpler symmetry $S_{\mu\tau,L} \otimes S_{\mu\tau,R}$, leads to a predictive $3+1$ picture for the LSND experiment which is consistent with all observations. As noted earlier, there are additional motivation to consider such symmetries.

We assume that the usual seesaw mechanism or more precisely the right handed neutrino mass matrix originates from an effective field theory whose ultraviolet completion contains the double seesaw form of the neutrino mass matrix [21] operating above the seesaw scale. The $S_{\mu\tau,L} \otimes S_{\mu\tau,R}$ is assumed to operate at this high scale. We then assume that there is a low scale symmetry breaking which gives mass to the sterile neutrino as well as its mixings with the active neutrinos. We find that in the absence of CP violation, as long as $S_{\mu\tau,L}$ symmetry is exact, we can predict the full 4×4 neutrino mass matrix. The active neutrinos

have an inverted hierarchy. We further point out that once we include CP violation, even though active neutrino hierarchy is inverted, there is no lower bound on the effective mass probed by the $\beta\beta_{0\nu}$ decay.

We also show that the same strategy can be extended to generate a natural $3+2$ model for LSND where one of the sterile neutrino is the right handed seesaw partner and another is a standard model singlet employed to implement the double seesaw scheme. In the same way as in the $3+1$ case, for the CP conserving case, here also can we predict all elements of the 5×5 neutrino mass matrix using existing fits to oscillation data.

II. THE BASIC IDEA AND THE $S_{\mu\tau,L} \otimes S_{\mu\tau,R}$ MODEL

The basic idea for getting the 3+1 model from the type I seesaw was outlined in Ref.[19]. The seesaw formula for light neutrino mass matrix is given by:

$$\mathcal{M}_\nu = -M_D^T M_R^{-1} M_D \tag{1}$$

Suppose we have a symmetry $\mathcal{S}$ in the theory that leads to $Det M_R = 0$ with only one of the eigenvalues zero. If the same symmetry also guarantees that $Det M_D = 0$, then one can use the seesaw formula. The way to proceed is to "take out" the zero mass eigen states from both M_R and M_D and then use the seesaw formula for the 2×2 system. It is then clear that the spectrum will consist of two light Majorana neutrinos (predominantly left-handed), two heavy right handed Majorana neutrinos; one massless right handed neutrino, which will play the role of the sterile neutrino of the 3+1 model and a massless left-handed neutrino. Breaking the symmetry $\mathcal{S}$ very weakly or by loop effects can lead to a nonzero mass for the sterile neutrino as well as its mixings with the $\nu_{e,\mu}$ so that one has an explanation of the LSND result via $\nu_e \to \nu_s \to \nu_\mu$ process. As already noted in [19], $\mathcal{S}$ was chosen to be $(L_e - L_\mu - L_\tau) \otimes S_{2R}$. Here we take a different path.

We consider a model based on the gauge group $SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$. In addition to the standard model matter we have $SU(2)_L$ singlets χ^c, N_i^c and S_i ($i = e, \mu, \tau$) with quantum numbers given in table 1 (we do not bring in the quark sector, whose gauge quantum numbers are obvious) . Here we have used the supersymmetric chiral field notation and dropped the chirality (L,R) indices. The superpotential involving these fields can be

written as follows:

$$W = h_\nu L H_u N^c + f N^c \chi^c S + \lambda \phi S S \qquad (2)$$

The couplings h_ν, f, λ are all 3×3 matrices in flavor space whose properties are constrained by the symmetry $S_{\mu\tau,L} \otimes S_{\mu\tau,R}$ as discussed below.

field	$SU(2)_L$	$U(1)_{I_{3R}}$	$U(1)_{B-L}$
$L_i \equiv (\nu, e)_i$	2	0	-1
N_i^c	1	$-\frac{1}{2}$	1
S_i	1	0	0
H_u	2	$\frac{1}{2}$	0
H_d	2	$-\frac{1}{2}$	0
χ^c	1	$\frac{1}{2}$	-1
ϕ	1	0	0

Under $S_{\mu\tau,L}$ symmetry, the left handed leptonic doublets (L_μ, L_τ) transform into each other; similarly under $S_{\mu\tau,R}$, (N_μ^c, N_τ^c) transform into each other. Invariance under this symmetry then restricts the matrices $M_D \equiv h_\nu v_u$ and $M \equiv f < \chi^c >$ as follows:

$$M_D = \begin{pmatrix} m_{11} & m_{12} & m_{12} \\ m_{21} & m_{22} & m_{22} \\ m_{21} & m_{22} & m_{22} \end{pmatrix} \qquad (3)$$

and

$$M = \begin{pmatrix} M_{11} & M_{12} & M_{12} \\ M_{21} & M_{22} & M_{23} \\ M_{21} & M_{22} & M_{23} \end{pmatrix} \qquad (4)$$

The matrix μ on the other hand has an arbitrary symmetric form. Our considerations are independent of whether the mass matrix elements are real or complex. We keep them complex to be quite general.

The superpotential of Eq. (2) leads to the following double seesaw form for the neutrino mass matrix at the scale $\mu = \lambda < \phi >\gg M \gg v_{wk}$:

$$\mathcal{M}_{\nu NS} = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_R \\ 0 & M_R^T & \mu \end{pmatrix} \qquad (5)$$

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The zeros in this mass matrix can be guaranteed by an extra $U(1)_X$ symmetry under which the fields N^c, L, S, ϕ and χ^c have charges $1, -1, 2, -4$ and -3 respectively. Note that we do not include a $\bar{\phi}$ superfield in the theory.

In order to obtain the light neutrino masses and mixings, we proceed in two steps: first, we decouple the S-fermions whose masses are above the scale M and write down the effective theory below that scale. The effective ν, N mass matrix below this mass scale can be written as:

$$\mathcal{M}_{\nu NS} = \begin{pmatrix} 0 & m_D \\ m_D^T & M\mu^{-1}M^T \end{pmatrix} \tag{6}$$

From Eq. (4), we see that determinant of the (N_e, N_μ, N_τ) mass matrix $M_R = M\mu^{-1}M^T$ vanishes and the zero mass eigenstate is given by $\frac{1}{\sqrt{2}}(N_\mu - N_\tau)$. We "take out" this state and the remaining right handed neutrino mass matrix is 2×2 involving only the states $(N_e, \frac{1}{\sqrt{2}}(N_\mu + N_\tau))$. Similarly, we can take out the state $\frac{1}{\sqrt{2}}(\nu_\mu - \nu_\tau)$ from the Dirac mass matrix m_D again leaving a 2×2 mass matrix in the basis as

$$(\nu_e, \frac{1}{\sqrt{2}}(\nu_\mu + \nu_\tau))\tilde{m}_D \begin{pmatrix} N_e \\ \frac{1}{\sqrt{2}}(N_\mu + N_\tau) \end{pmatrix} \tag{7}$$

As already noted the seesaw matrix is now a two generation matrix giving two light Majorana neutrino states. The other active light neutrino state is the massless state $\frac{1}{\sqrt{2}}(\nu_\mu - \nu_\tau)$. The light sterile neutrino state is the massless state $\nu_s \equiv \frac{1}{\sqrt{2}}(N_\mu - N_\tau)$. This therefore provides the starting ingredient of the 3+1 sterile neutrino model.

III. SYMMETRY BREAKING, MASSES AND MIXINGS OF THE STERILE NEUTRINO

Having explained why the sterile neutrino is light, we now proceed to make it a realistic model for LSND by putting a mass term of order eV for ν_s and smaller off diagonal mixings for ν_s with the active neutrinos. These small scales could arise from a scalar field having a keV scale vev as in Ref.[22] which can also help us to avoid the WMAP bound[23].

So in the basis $(\nu_e, \nu_+, \nu_-, \nu_s)$, we have a mass matrix of the form

$$\mathcal{M}_{\nu,\nu_s,\alpha\beta} = m_{\alpha\beta} \tag{8}$$

There are two cases that one can consider: (i) only $S_{\mu\tau,R}$ is broken so that the massless state ν_- remains decoupled and we have a 3×3 mass matrix with involving (ν_e, ν_+, ν_s) and (ii)

when all four states are mixed and both the interchange symmetries are broken. The second case is similar to a recent analysis in Ref.[18].

A. Case (i): unbroken $S_{\mu\tau,L}$

The light neutrino mass matrix in this case is a 4×4 matrix which in weak (flavor) basis can be written as

$$
M_\nu = \begin{pmatrix} m_{11} & m_{12} & m_{12} & f_1 \\ m_{12} & m_{22} & m_{22} & f_2 \\ m_{12} & m_{22} & m_{22} & f_2 \\ f_1 & f_2 & f_2 & f_4 \end{pmatrix}.
\tag{9}
$$

Equality of certain entries in this matrix is a reflection of the unbroken symmetry. This matrix in general has six real parameters and three complex phases. To diagonalize this matrix, we first note that without mixing terms f_1, f_2, the diagonalizing orthogonal matrix is given by

$$
U^{(0)} = \begin{pmatrix} c & s & 0 & 0 \\ \frac{-s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\tag{10}
$$

where $s \equiv \sin\theta, c \equiv \theta$,and θ is solar mixing angle and eignevalues

$$
\begin{aligned}
m_1^{(0)} &= \frac{m_{11} + 2m_{22}}{2} - \frac{1}{2}\sqrt{8m_{12}^2 + (2m_{22} - m_{11})^2} \\
m_2^{(0)} &= \frac{m_{11} + 2m_{22}}{2} + \frac{1}{2}\sqrt{8m_{12}^2 + (2m_{22} - m_{11})^2} \\
m_3^{(0)} &= 0 \\
m_4^{(0)} &= f_4
\end{aligned}
\tag{11}
$$

Note that the vanishing of the mass of the third eigenstate is due to unbroken $S_{\mu\tau,L}$ symmetry. This case has inverted hierarchy for active neutrino masses.

In the limit where the sterile neutrino decouples, $\mu - \tau$ symmetry would have dictated that we have four parameters describing the 3×3 neutrino mass matrix. However, in our case, the effective theory has a higher symmetry $M_\nu = M_\nu S_{\mu\tau,L} = S_{\mu\tau,L} M_\nu$. As a

result, since there are three experimental inputs i.e. $\Delta m_\odot^2$, Δm_{atm}^2 and $\sin^2\theta_\odot$, the three neutrino mass matrix in our case is completely fixed by data. Using the best fit values $\Delta m_{atm}^2 = 2.2 \times 10^{-3} eV^2$, $\Delta m_\odot^2 = 7.9 \times 10^{-5} eV^2$ and $\sin^2\theta_\odot = 0.30$ [24],the mass matrix of active neutrinos is given by

$$M_\nu = \begin{pmatrix} 0.0471547 & 0.000270318 & 0.000270318 \\ 0.000270318 & 0.0237444 & 0.0237444 \\ 0.000270318 & 0.0237444 & 0.0237444 \end{pmatrix} (eV). \tag{12}$$

We have ignored CP violation in this discussion. From Eq.(12), we can read off the value for the effective neutrino mass $< m_{ee} >$ probed by $\beta\beta_{0\nu}$ experiment. It is nothing but the ee entry in Eq.(12) and we find that the central value of $< m_{ee} > \simeq 47$ meV. In this decoupled case, of course, we cannot accomodate the LSND results since the decoupling essentially amounts to very small mixings.

Including the effect of the small terms $f_1, f_2 \ll f_4$ so that LSND mixings are accomodated in the picture as originally intended,leads to the following form for the generalized 4×4 PMNS matrix.

$$U \simeq \begin{pmatrix} c & s & 0 & \delta_1 \\ \frac{-s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \delta_2 \\ \frac{-s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & \delta_2 \\ -\delta_1 c + \sqrt{2}\delta_2 s & -\delta_1 s - \sqrt{2}\delta_2 c & 0 & 1 \end{pmatrix} \tag{13}$$

where

$$\delta_1 \simeq \frac{f_1}{f_4}; \delta_2 \simeq \frac{f_2}{f_4}$$

$$\tag{14}$$

The mass eigenvalues now get shifted to the values $m_1 - \frac{f_1^2}{f_4}$ and $m_2 - \frac{f_2^2}{f_4}$. LSND results require that $\frac{f_{1,2}}{f_4} \simeq 0.2$. Naturalness would require that $\frac{f_2^2}{f_4} \simeq 0.01$ implying that $f_{1,2} \simeq 0.05$ eV. A detailed numerical analysis of this which fits the central values of all observed parameters is given below.

Unlike the case when the sterile neutrino is decoupled, here we have six parameters describing the four neutrino mass matrix in the absence of CP violation. All the elements of the mass matrix can therefore be determined using $\Delta m_\odot^2$, Δm_{atm}^2 and $\sin^2\theta_\odot$ from [24] as

well as Δm^2_{LSND}, $|U_{e4}|$ and $|U_{e\mu}|$[14]. We have used $\Delta m^2_{LSND} = 0.92 eV^2$, $|U_{e4}| = 0.136$ and $|U_{\mu4}| = 0.205$ and find (ignoring CP phases) that the four neutrino mass matrix is given by

$$
M_\nu = \begin{pmatrix}
0.06214 & 0.0240057 & 0.0240057 & 0.117 \\
0.0240057 & 0.0612706 & 0.0612706 & 0.18441 \\
0.0240057 & 0.0612706 & 0.0612706 & 0.18441 \\
0.117 & 0.18441 & 0.18441 & 0.955
\end{pmatrix} (eV) \tag{15}
$$

We see that in the absence of CP violation, the value of $< m_{ee} > \simeq 62$ meV. This is accessible to the proposed searches for neutrinoless double beta decay[25].

B.   Case (ii)

We can extend U to the more general case where the $S_{\mu\tau,L}$ symmetry is also broken. In this case, the neutrino mass matrix has all entries arbitrary except that f_4 is the largest entry in it. Since $S_{\mu\tau,L}$ is broken weakly, the U matrix can be written as

$$
U = \begin{pmatrix}
c & s & \frac{\theta_{13}}{\sqrt{2}} & \delta_1 \\
\frac{-s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \delta_2 \\
\frac{-s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & \delta_3 \\
-\delta_1 c + \frac{(\delta_2+\delta_3)s}{\sqrt{2}} & -\delta_1 s - \frac{(\delta_1+\delta_2)c}{\sqrt{2}} & \frac{-\delta_2+\delta_3-\delta_1\theta_{13}}{\sqrt{2}} & 1
\end{pmatrix} \tag{16}
$$

θ_{13} in this case will be proportional to the small $S_{\mu\tau,L}$ breaking. However, in this case, even in the absence of CP violation, one cannot predict the neutrino matrix fully.

IV.   PHENOMENOLOGICAL IMPLICATIONS

An interesting implication of the sterile neutrino is its impact on the neutrinoless double beta decay. As we saw in the previous section, if CP phases are completely ignored, we get a definite prediction for the effective double beta decay mass $< m_{ee} >$ which is the ee entry of the neutrino mass matrix. Once CP violation is included, the situation changes and we have the effective electron neutrino mass given by

$$
< m_{ee} > \simeq |c^2 m_1 + s^2 e^{2i\alpha} m_2 + |U_{e4}|^2 e^{2i\beta} m_4| \simeq |\sqrt{\Delta m^2_{atm}}(c^2 + s^2 e^{2i\alpha}) + |U_{e4}|^2 e^{2i\beta}\sqrt{\Delta m^2_{LSND}}| \tag{17}
$$

where $\Delta m^2_{atm} = m_2^2$ and $\Delta m^2_{LSND} \simeq m_4^2$ are the atmospheric and the LSND mass squared differences . Note that which $< m_{ee} >$ can be large (if there is no accidental cancellation as

was the case in Eq. (15)) with

$$< m_{ee} >^{max} = \sqrt{\Delta m^2_{atm}} + |U_{e4}|^2 \sqrt{\Delta m^2_{LSND}} \simeq 62 \; meV \qquad (18)$$

In figure 1 we present a scatter plot for $< m_{ee} >$ as function of $|U_{e4}|^2 \sim \delta_1^2$. One can see that there is a parameter range where the rate of $(\beta\beta)_{0\nu}$ process almost vanishes, which implies that there is CP violation due to sterile neutrino sector.
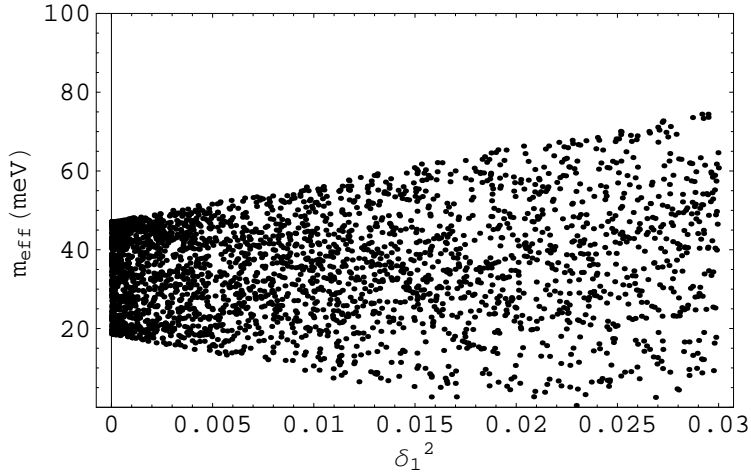

FIG. 1: Scatter plot of $|m_{eff}| \equiv < m_{ee} >$ in the case of inverted hierarchy in the presence of a sterile neutrino with mass ~ 1 eV as a function of δ_1^2. Note that the minimum value of $|m_{eff}|$ depends on the best fit value for the LSND mixing parameter in a 3+1 scheme. The scatter of points is due to the variation of the phases α and β between 0 to π.

Thus the well known lower bound on neutrinoless double beta decay in the case of inverted hierarchy[26] disappears once the effect of sterile neutrino is included. In other words, if Mini-BooNe confirms LSND results, a null result in the next round of double beta decay experiments will not necessarily rule out the inverted hierarchy scenario for neutrinos nor the Majorana nature of the neutrino. For example, in the absence of the sterile neutrino, if there was no signal for $\beta\beta_{0\nu}$ decay at the level of 30 meV and if long base line experiments confirmed that neutrinos have inverted hierarchy, one would have concluded that the neutrino could be a Dirac fermion. The presence of the sterile neutrino would clearly change this conclusion. This is an important effect of the presence of the LSND sterile neutrino.

We also note that the neutrino mass inferred from the study of the shape of the electron

energy spectrum near the end point of radioactive nuclear decay is

$$m_\beta = \sum_i |U_{ei}|^2 m_i$$

$$\simeq \sqrt{\Delta m_{atm}^2} + |U_{e4}|^2 \sqrt{\Delta m_{LSND}^2} \simeq 62 \; meV \tag{19}$$

This is below the present expectations in the Katrin experiment[27] but is of interest for future studies in this area.

V. EXTENSION TO $3+2$ CASE

In this section, we discuss how our strategy can be extended to obtain a $3+2$ model for LSND. There are several ways to achieve this. The essential new step is to extend the discrete symmetry to $S_{\mu\tau,L} \times S_{\mu\tau,R} \times S_{\mu\tau,S}$ and introduce an additional singlet fermion. This implies that in the symmetry limit only the combination $S_2 + S_3$ field has mass and there are two massless sterile neutrinos: $N_\mu^c - N_\tau^c$ and $S_\mu - S_\tau$. We can now give mass to them as well as mixings by including the Majorana mass terms of type $\mu S_a S_b$ where μ is in the eV range, that break $S_{\mu\tau,R} \times S_{\mu\tau,S}$ part of the discrete group while keeping $S_{\mu\tau,L}$ unbroken. The form of the mass matrix in this case is given by

$$\mathcal{M}_{\nu NS} = \begin{pmatrix} 0 & m_D & 0 & 0 \\ m_D^T & 0 & M_{RS} & 0 \\ 0 & M_{RS}^T & 0 & M_{ST} \\ 0 & 0 & M_{ST} & \mu \end{pmatrix} \tag{20}$$

This reduces to the form in Eq. (5) as μ becomes large giving two light active neutrinos via the double seesaw as discussed previously.

With 2 sterile neutrinos with $\mu - \tau$ symmetry, the general mass matrix can be written as

$$M_\nu = \begin{pmatrix} m_{11} & m_{12} & m_{12} & f_1 & g_1 \\ m_{12} & m_{22} & m_{22} & f_2 & g_2 \\ m_{12} & m_{22} & m_{22} & f_2 & g_2 \\ f_1 & f_2 & f_2 & f_4 & g_3 \\ g_1 & g_2 & g_2 & g_3 & g_4 \end{pmatrix} \tag{21}$$

The parameter g_3 can be set to zero by a rotation of the two sterile neutrinos leaving a total of nine parameters. Compared with 3+1 scheme, we have three extra parameters, g_1, g_2, g_4.

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From the experiment data fit,we have $\Delta m_{41}^2, |U_{e4}|, |U_{\mu4}|, \Delta m_{51}^2, |U_{e5}|, |U_{\mu5}|$[14], which has three more inputs, we can determine the full mass matrix (in the absence of any CP phases). Now the 5×5 PMNS matrix U is given by

$$
U \simeq \begin{pmatrix}
c & s & 0 & \delta_1 & \epsilon_1 \\
\frac{-s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \delta_2 & \epsilon_2 \\
\frac{-s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & \delta_2 & \epsilon_2 \\
-\delta_1 c + \sqrt{2}\delta_2 s & -\delta_1 s - \sqrt{2}\delta_2 c & 0 & 1 & 0 \\
-\epsilon_1 c + \sqrt{2}\epsilon_2 s & -\epsilon_1 s - \sqrt{2}\epsilon_2 c & 0 & 0 & 1
\end{pmatrix}
\tag{22}
$$

All the elements of the mass matrix can be determined using $\Delta m_\odot^2, \Delta m_{atm}^2$ and $\sin^2\theta_\odot$ as well as $\Delta m_{41}^2, |U_{e4}|, |U_{\mu4}|, \Delta m_{51}^2, |U_{e5}|, |U_{\mu5}|$[14]. Corresponding to the two solutions in [14] we get two solutions for the mass matrix.

With the best fit data (i) $\Delta m_{41}^2 = 0.92 eV^2, |U_{e4}| = 0.121, |U_{\mu4}| = 0.204, \Delta m_{51}^2 = 22 eV^2, |U_{e5}| = 0.036, |U_{\mu5}| = 0.224$[14].

$$
M_v = \begin{pmatrix}
0.0672934 & 0.0618002 & 0.0618002 & 0.110382 & 0.167045 \\
0.0618002 & 0.299067 & 0.299067 & 0.186183 & 1.04006 \\
0.0618002 & 0.299067 & 0.299067 & 0.186183 & 1.04006 \\
0.110382 & 0.186183 & 0.186183 & 0.964982 & 0.00456411 \\
0.167045 & 1.04006 & 1.04006 & 0.00456411 & 4.69549
\end{pmatrix} (eV)
\tag{23}
$$

With the best fit data (ii) $\Delta m_{41}^2 = 0.46 eV^2, |U_{e4}| = 0.090, |U_{\mu4}| = 0.226, \Delta m_{51}^2 = 0.89 eV^2, |U_{e5}| = 0.125, |U_{\mu5}| = 0.160$.

$$
M_v = \begin{pmatrix}
0.06742 & 0.0329899 & 0.0329899 & 0.0568206 & 0.11209 \\
0.0329899 & 0.0826493 & 0.0826493 & 0.14289 & 0.143498 \\
0.0329899 & 0.0826493 & 0.0826493 & 0.14289 & 0.143498 \\
0.0568206 & 0.14289 & 0.14289 & 0.685108 & 0.00398793 \\
0.11209 & 0.143498 & 0.143498 & 0.00398793 & 0.947753
\end{pmatrix} (eV)
\tag{24}
$$

In both the cases, there is a prediction for $m_{ee} \simeq 67$ meV, which is very similar to the $3+1$ case without CP violation. Once CP violating phases are included, there is an effect on m_{ee}. The effects for both cases are again similar to the $3+1$ case.

VI. SUMMARY

In summary, we have presented an example where using a simple discrete symmetry, one gets one of the seesaw right handed neutrinos to play the role of a light sterile neutrino needed to understand the LSND results. The discrete symmetries used are experimentally motivated by the near maximal value of the atmospheric mixing angle. The model not only can accomodate the LSND results, but we find that due to the symmetries used to generate the light sterile neutrino, all elements of the neutrino mass matrix are determined by present data if CP phases are ignored. Once CP violation is included, the lower bound on the effective mass for neutrinoless double beta decay known for the three neutrino case with inverted hierarchy is eliminated. Thus confirmation of LSND results by Mini-BooNe experiment will not only revolutionize our thinking about the nature of new physics beyond the standard model (by requiring the concepts such as mirror neutrinos or leptonic symmetries or something else) but it will also affect our conclusions about whether the neutrino is a Majorana or Dirac fermion. We further show that our strategy and analysis can be very simply extended to the $3 + 2$ scenario for LSND, with similar results.

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